Chapter 15
Optimization Modeling: Applications
Integer Programming

Introduction
• When one or more variables in an LP problem must assume an integer value we have an Integer Linear Programming (ILP) problem.
• ILPs occur frequently
  – Scheduling
  – Manufacturing
• Integer variables also allow us to build more accurate models for a number of common business problems.

Three Types of ILP Models
• All Integer Model
• 0-1 Integer Model (Binary Variables)
• Mixed Integer Model

Manufacturing Example
• Machine shop is expanding and plans to buy lathes and presses
• Owner estimates that profit will be $100/day for each press and each lathe will yield a profit of $150/day
• Purchase limit by cost & floor space are:

<table>
<thead>
<tr>
<th>Machine</th>
<th>Floor Space</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Press</td>
<td>15</td>
<td>$8,000</td>
</tr>
<tr>
<td>Lathe</td>
<td>30</td>
<td>$4,000</td>
</tr>
</tbody>
</table>
• Budget of $40,000 and 200 ft² floor space
**Integrality Conditions**

Max $Z = 100X_1 + 150X_2$ } profit
subject to:

- $15X_1 + 30X_2 \leq 200$ ft$^2$ } floor space
- $8,000X_1 + 4,000X_2 \leq 40,000$ } Price
- $X_1, X_2 \geq 0$ } nonnegativity
- $X_1, X_2$ must be integers } integrality

Integrality conditions are easy to state but make the problem much more difficult (and sometimes impossible) to solve.

**Relaxation**

- LP Model Relaxing Integrality Condition
  Max $Z = 100X_1 + 150X_2$
  subject to
  $15X_1 + 30X_2 \leq 200$ ft$^2$
  $8,000X_1 + 4,000X_2 \leq 40,000$
  $X_1, X_2 \geq 0$

**Integer Feasible vs. LP Feasible Region**

Solving ILP Problems

- When solving an Integer Linear Programming model by relaxing the integrality conditions, sometimes you “get lucky” and obtain an integer feasible solution.
**Bounds**

- The optimal solution to an LP relaxation of an ILP problem gives us a *bound* on the optimal objective function value.
- For *maximization* problems, the optimal relaxed objective function values is an *upper bound* on the optimal integer value.
- For *minimization* problems, the optimal relaxed objective function values is a *lower bound* on the optimal integer value.

**Rounding**

- It is tempting to simply round a fractional solution to the closest integer solution.
- In general, this does not work reliably. Why?
  - The rounded solution may be infeasible.
  - The rounded solution may be suboptimal.

**How Rounding Down Can Result in an Infeasible Solution**

![Diagram showing how rounding down can result in an infeasible solution](image)

**Branch-and-Bound**

- The Branch-and-Bound (B&B) algorithm can be used to solve ILP problems.
- Requires the solution of a series of LP problems termed “candidate problems”.
- Theoretically, this can solve any ILP.
- Practically, it often takes *LOTS* of computational effort (and time).
**Stopping Rules**

- Because B&B takes so long, most ILP packages allow you to specify a suboptimality tolerance factor.
- This allows you to stop once an integer solution is found that is within some %-age of the global optimal solution.
- Bounds obtained from LP relaxations are helpful here.
  - Example
    - LP relaxation has an optimal obj. value of $1,055.56$
    - 95% of $1,055.56$ is $1,002.78$.
    - Thus, an integer solution with obj. value of $1,002.78$ or better must be within 5% of the optimal solution.

**When Using Solver**

- Set decision variables to “int” value in constraint dialog box
- Solver has default tolerance of 5%; Change to 0
- Solver will give you the GLOBAL OPTIMAL solution

**Binary Variables**

- Binary variables are integer variables that can assume only two values: 0 or 1.
- These variables can be useful in a number of practical modeling situations….
  - Shortest route/path problem
  - Scheduling/assignment problems
  - Networking problems

**Software Note**

- Solver has a “bin” option for identifying binary variables.
- You will find this “bin” option with the “int” option in the dropdown box used when adding constraints.
A Community Council Problem: Building Facilities

A community council must decide which recreation facilities to construct in its community. Four new recreation facilities have been proposed — a swimming pool, a tennis center, an athletic field, and a gymnasium. The council wants to construct facilities that will maximize the expected daily usage by the residents of the community subject to land and cost limitations. The expected daily usage and cost and land requirements for each facility are shown below.

<table>
<thead>
<tr>
<th>Facility</th>
<th>Usage</th>
<th>Cost</th>
<th>Land</th>
</tr>
</thead>
<tbody>
<tr>
<td>Swimming Pool</td>
<td>300</td>
<td>$35,000</td>
<td>4</td>
</tr>
<tr>
<td>Tennis center</td>
<td>90</td>
<td>$10,000</td>
<td>2</td>
</tr>
<tr>
<td>Athletic Field</td>
<td>400</td>
<td>$25,000</td>
<td>7</td>
</tr>
<tr>
<td>Gymnasium</td>
<td>150</td>
<td>$90,000</td>
<td>3</td>
</tr>
</tbody>
</table>

The community has a $120,000 construction budget and 12 acres of land. Because the land for the swimming pool and tennis center are in the same area of the community, however, only one of these two facilities can be constructed. The council wants to know which of the recreation facilities to construct in order to maximize the expected daily usage.

Formulate a model for this problem

Mixed Integer LP Model

- This problem is one in which some, but not all, of the decision variables must be integers
**Investment Example**

Nancy Smith has $250,000 to invest in three alternative investments — condominiums, land, and municipal bonds. She wants to invest in the alternatives that will result in the greatest return on investment after one year. Each condominium costs $50,000 and will return a profit of $9,000 if sold at the end of one year; each acre of land costs $12,000 and will return a profit of $1,500 at the end of one year; and each municipal bond costs $8,000 and will result in a return of $1,000 if sold at the end of one year. In addition, there are only 4 condominiums, 15 acres of land, and 20 municipal bonds available for purchase. What investment plan would you recommend to Nancy? Formulate a linear programming model and solve.

**The Fixed-Charge Problem**

- Many decisions result in a fixed or lump-sum cost being incurred:
  - The cost to lease, rent, or purchase a piece of equipment or a vehicle that will be required if a particular action is taken.
  - The setup cost required to prepare a machine or production line to produce a different type of product.
  - The cost to construct a new production line or facility that will be required if a particular decision is made.
  - The cost of hiring additional personnel that will be required if a particular decision is made.

**Example Fixed-Charge Problem: Remington Manufacturing**

Remington Manufacturing is planning its next production cycle. The company can produce three products, each of which must undergo machining, grinding, and assembly operations. The table below summarizes the hours of machining, grinding, and assembly required by each unit of each product, and the total hours of capacity available for each operation.
**Example Fixed-Charge Problem:**

**Remington Manufacturing**

<table>
<thead>
<tr>
<th>Operation</th>
<th>Prod. 1</th>
<th>Prod. 2</th>
<th>Prod. 3</th>
<th>Hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>Machining</td>
<td>2</td>
<td>3</td>
<td>6</td>
<td>600</td>
</tr>
<tr>
<td>Grinding</td>
<td>6</td>
<td>3</td>
<td>4</td>
<td>300</td>
</tr>
<tr>
<td>Assembly</td>
<td>5</td>
<td>6</td>
<td>2</td>
<td>400</td>
</tr>
</tbody>
</table>

Unit Profit: $48, $55, $50

Setup Cost: $1000, $800, $900

---

**Defining the Decision Variables**

\[ X_i = \text{the amount of product } i \text{ to be produced, } i = 1, 2, 3 \]

\[ Y_i = \begin{cases} 
1 & \text{if } X_i > 0 \\
0, \text{if } X_i = 0 & \text{if } i = 1, 2, 3
\end{cases} \]

---

**Defining the Objective Function**

Maximize total profit.

\[ \text{MAX: } 48X_1 + 55X_2 + 50X_3 - 1000Y_1 - 800Y_2 - 900Y_3 \]

---

The cost accounting department has estimated that each unit of product 1 manufactured and sold will contribute $48 to profit, and each unit of products 2 and 3 contributes $55 and $50, respectively. However, manufacturing a unit of product 1 requires a setup operation on the production line that costs $1,000. Similar setups are required for products 2 and 3 at costs of $800 and $900, respectively. The marketing department believes it can sell all the products produced. Therefore, the management of Remington wants to determine the most profitable mix of products to produce.
Defining the Constraints

- Resource Constraints
  2X₁ + 3X₂ + 6X₃ \leq 600 \} machining
  6X₁ + 3X₂ + 4X₃ \leq 300 \} grinding
  5X₁ + 6X₂ + 2X₃ \leq 400 \} assembly

- Binary Constraints
  All Yᵢ must be binary

- Nonnegativity conditions
  Xᵢ \geq 0, \ i = 1, 2, ..., 3

- Is there a missing link?

Defining the Constraints (cont’d)

- Linking Constraints (with “Big M”)
  X₁ \leq M₁Y₁ \ or \ X₁ - M₁Y₁ \leq 0
  X₂ \leq M₂Y₂ \ or \ X₂ - M₂Y₂ \leq 0
  X₃ \leq M₃Y₃ \ or \ X₃ - M₃Y₃ \leq 0

  If Xᵢ > 0 these constraints force the associated Yᵢ to equal 1.
  If Xᵢ = 0 these constraints allow Yᵢ to equal 0 or 1, but the objective will cause Solver to choose 0.

  Note that Mᵢ imposes an upper bounds on Xᵢ.

  It helps to find reasonable values for the Mᵢ.

Finding Reasonable Values for M₁

- Consider the resource constraints
  2X₁ + 3X₂ + 6X₃ \leq 600 \} machining
  6X₁ + 3X₂ + 4X₃ \leq 300 \} grinding
  5X₁ + 6X₂ + 2X₃ \leq 400 \} assembly

- What is the maximum value X₁ can assume?
  Let X₂ = X₃ = 0
  X₁ = \text{MAX}(600/2, 300/6, 400/5)
  = \text{MAX}(300, 50, 80)
  = 300

- Maximum values for X₂ & X₃ can be found similarly.

Summary of the Model

MAX: 48X₁ + 55X₂ + 50X₃ - 1000Y₁ - 800Y₂ - 900Y₃
Subject to:
  2X₁ + 3X₂ + 6X₃ \leq 600 \} machining
  6X₁ + 3X₂ + 4X₃ \leq 300 \} grinding
  5X₁ + 6X₂ + 2X₃ \leq 400 \} assembly
  X₁ - 300Y₁ \leq 0
  X₂ - 200Y₂ \leq 0
  X₃ - 200Y₃ \leq 0

  \} linking

All Yᵢ must be binary
Xᵢ \geq 0, \ i = 1, 2, 3
Implementing the Model

Potential Pitfall

- **DO NOT** use IF() functions to model the relationship between the $X_i$ and $Y_i$
  - Suppose cell C7 represents $X_i$
  - Suppose cell C17 represents $Y_i$
  - You’ll want to let C17 = IF(C7>0,1,0)
  - This **WILL NOT WORK** with Solver!
- Treat the $Y_i$ just like any other variable.
  - Make them changing cells.
  - Use the linking constraints to enforce the proper relationship between the $X_i$ and $Y_i$

Nonlinear Programming

- An NLP problem has a nonlinear objective function and/or one or more nonlinear constraints.
- NLP problems are formulated and implemented in virtually the same way as linear problems.
- The mathematics involved in solving NLPs is quite different than for LPs.
- Solver tends to mask this difference but it is important to understand the difficulties that may be encountered when solving NLPs.
Possible Optimal Solutions to NLPs
(not occurring at corner points)

- Linear objective, nonlinear constraints
- Nonlinear objective, linear constraints
- Nonlinear objective, nonlinear constraints

The GRG Algorithm

- Solver uses the Generalized Reduced Gradient (GRG) algorithm to solve NLPs.
- GRG can also be used on LPs but is slower than the Simplex method.
- The following discussion gives a general (but somewhat imprecise) idea of how GRG works.

An NLP Solution Strategy

- Local vs. Global Optimal Solutions
Comments About NLP Algorithms

• It is not always best to move in the direction producing the fastest rate of improvement in the objective.
• NLP algorithms can terminate a local optimal solutions.
• The starting point influences the local optimal solution obtained.

Comments About Starting Points

• The null starting point should be avoided.
• When possible, it is best to use starting values of approximately the same magnitude as the expected optimal values.

A Note About “Optimal” Solutions

• When solving a NLP problem, Solver normally stops when the first of three numerical tests is satisfied, causing one of the following three completion messages to appear:
  1) “Solver found a solution. All constraints and optimality conditions are satisfied.”
     This means Solver found a local optimal solution, but does not guarantee that the solution is the global optimal solution. Unless you know that a problem has only one local optimal solution (which must also be the global optimal), you should run Solver from several different starting points to increase the chances that you find the global optimal solution to your problem.
  2) “Solver has converged to the current solution. All constraints are satisfied.”
     This means the objective function value changed very slowly for the last few iterations. If you suspect the solution is not a local optimal, your problem may be poorly scaled. In Excel 8.0, the convergence option in the Solver Options dialog box can be reduced to avoid convergence at suboptimal solutions.
  3) “Solver cannot improve the current solution. All constraints are satisfied.”
     This rare message means the your model is degenerate and the Solver is cycling. Degeneracy can often be eliminated by removing redundant constraints in a model.

The Economic Order Quantity (EOQ) Problem

• Involves determining the optimal quantity to purchase when orders are placed.
• Small orders result in:
  – low inventory levels & carrying costs
  – frequent orders & higher ordering costs
• Large orders result in:
  – higher inventory levels & carrying costs
  – infrequent orders & lower ordering costs
**Sample Inventory Profiles**

<table>
<thead>
<tr>
<th>Month</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual Usage</td>
<td>150</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Order Size</td>
<td>50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Avg Inventory</td>
<td>25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**The EOQ Model**

Total Annual Cost = \(DC + \frac{DS}{Q} + \frac{Q}{2}Ci\)

where:
- \(D\) = annual demand for the item
- \(C\) = unit purchase cost for the item
- \(S\) = fixed cost of placing an order
- \(i\) = cost of holding inventory for a year (expressed as a % of \(C\))
- \(Q\) = order quantity

Assumes:
- Demand (or use) is constant over the year
- New orders are received in full when the inventory level drops to zero.

**EOQ Cost Relationships**

**An EOQ Example: Ordering Paper For MetroBank**

  - Annual demand (\(D\)) is for 24,000 boxes
  - Each box costs $35 (\(C\))
  - Each order costs $50 (\(S\))
  - Inventory carrying costs are 18% (\(i\))
- What is the optimal order quantity (\(Q\))?
The Model

MIN: DC + \frac{D}{Q} S + \frac{Q}{2} Ci

Subject to: Q \geq 1

(Note the nonlinear objective!)

Implementing the Model

<table>
<thead>
<tr>
<th>MetroBank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual Demand</td>
</tr>
<tr>
<td>Cost per Unit</td>
</tr>
<tr>
<td>Cost per Order</td>
</tr>
<tr>
<td>Holding Cost</td>
</tr>
<tr>
<td>Order Quantity</td>
</tr>
<tr>
<td>Purchasing Cost</td>
</tr>
<tr>
<td>Cost of Ordering</td>
</tr>
<tr>
<td>Inventory Cost</td>
</tr>
<tr>
<td>Total Cost</td>
</tr>
</tbody>
</table>

Comments on the EOQ Model

• Using calculus, it can be shown that the optimal value of Q is:

\[ Q^* = \sqrt{\frac{2DS}{Ci}} \]

• Numerous variations on the basic EOQ model exist accounting for:
  – quantity discounts
  – storage restrictions
  – backlogging
  – etc

Location Problems

• Many decision problems involve determining optimal locations for facilities or service centers. For example,
  – Manufacturing plants
  – Warehouse
  – Fire stations
  – Ambulance centers
• These problems usually involve distance measures in the objective and/or constraints.

• The straight line (Euclidean) distance between two points \((X_1, Y_1)\) and \((X_2, Y_2)\) is:
**A Location Problem: Rappaport Communications**

- Rappaport Communications provides cellular phone service in several mid-western states.
- They want to expand to provide inter-city service between four cities in northern Ohio.
- A new communications tower must be built to handle these inter-city calls.
- The tower will have a 40 mile transmission radius.

**Defining the Decision Variables**

\[ X_1 = \text{location of the new tower with respect to the X-axis} \]

\[ Y_1 = \text{location of the new tower with respect to the Y-axis} \]

**Defining the Objective Function**

- Minimize the total distance from the new tower to the existing towers

\[
\text{MIN: } \sqrt{(5 - X_1)^2 + (45 - Y_1)^2} + \sqrt{(12 - X_1)^2 + (21 - Y_1)^2} \\
+ \sqrt{(17 - X_1)^2 + (5 - Y_1)^2} + \sqrt{(52 - X_1)^2 + (21 - Y_1)^2}
\]
Defining the Constraints

- Cleveland: \( \sqrt{(5 - X_1)^2 + (45 - Y_1)^2} \leq 40 \)
- Akron: \( \sqrt{(12 - X_2)^2 + (21 - Y_2)^2} \leq 40 \)
- Canton: \( \sqrt{(17 - X_3)^2 + (5 - Y_3)^2} \leq 40 \)
- Youngstown: \( \sqrt{(62 - X_4)^2 + (21 - Y_4)^2} \leq 40 \)

Implementing the Model

<table>
<thead>
<tr>
<th>Coordinates</th>
<th>Distance to Tower</th>
<th>Maximum Allowed</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tower</strong></td>
<td><strong>X</strong></td>
<td><strong>Y</strong></td>
</tr>
<tr>
<td>Cleveland</td>
<td>5</td>
<td>46</td>
</tr>
<tr>
<td>Akron</td>
<td>12</td>
<td>21</td>
</tr>
<tr>
<td>Canton</td>
<td>17</td>
<td>5</td>
</tr>
<tr>
<td>Youngstown</td>
<td>52</td>
<td>21</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Analyzing the Solution

- The optimal location of the “new tower” is in virtually the same location as the existing Akron tower.
- Maybe they should just upgrade the Akron tower.
- The maximum distance is 39.8 miles to Youngstown.
- This is pressing the 40 mile transmission radius.
- Where should we locate the new tower if we want the maximum distance to the existing towers to be minimized?
**Comments on Location Problems**

- The optimal solution to a location problem may not work:
  - The land may not be for sale.
  - The land may not be zoned properly.
  - The “land” may be a lake.

- In such cases, the optimal solution is a good starting point in the search for suitable property.

- Constraints may be added to location problems to eliminate infeasible areas from consideration.

**Mixture Problem**

A metal works manufacturing company produces four products fabricated from sheet metal in a plant that consists of four operations – stamping, assembly, finishing, and packaging. The processing times per unit for each operation and total available hourly per month are as follows:

<table>
<thead>
<tr>
<th>Product (Units)</th>
<th>Operation 1</th>
<th>Operation 2</th>
<th>Operation 3</th>
<th>Operation 4</th>
<th>Total Processing Time (hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stamping</td>
<td>Assembly</td>
<td>Finishing</td>
<td>Packaging</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.07</td>
<td>0.15</td>
<td>0.38</td>
<td>0.12</td>
<td>0.72</td>
</tr>
<tr>
<td>2</td>
<td>0.08</td>
<td>0.19</td>
<td>0.33</td>
<td>0.12</td>
<td>0.72</td>
</tr>
<tr>
<td>3</td>
<td>0.12</td>
<td>0.16</td>
<td>0.31</td>
<td>0.12</td>
<td>0.72</td>
</tr>
<tr>
<td>4</td>
<td>0.07</td>
<td>0.15</td>
<td>0.38</td>
<td>0.12</td>
<td>0.72</td>
</tr>
</tbody>
</table>

The sheet metal required for each product, the maximum demand per month, the maximum required contracted production, and the profit per product are as follows:

<table>
<thead>
<tr>
<th>Product</th>
<th>Sheet Metal (sq ft)</th>
<th>Minimum Demand</th>
<th>Maximum Demand</th>
<th>Profit ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.1</td>
<td>300</td>
<td>3,000</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>4.5</td>
<td>400</td>
<td>4,000</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>3.2</td>
<td>300</td>
<td>3,000</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>3.2</td>
<td>300</td>
<td>3,000</td>
<td>7</td>
</tr>
</tbody>
</table>

The company has 5,200 square feet of fabricated metal available each month. Formulate a linear programming model for this problem and solve the model using the computer.

**Diet Problem**

Bob and Mary Deere own and operate Brookside Farm. They raise cattle, horses, and chickens on the farm and also grow a variety of crops. The Deeres want to determine an optimal feed mix for each type of livestock they have at the minimum cost. They have five ingredients they either grow or obtain from the local feed store – corn, soybeans, fish meal, oats, and limestone nutrient mix. The number of milligrams of nutrients provided per pound of each ingredient and the cost of each ingredient are provided in the following table:

<table>
<thead>
<tr>
<th>Nutrient (mg)</th>
<th>Ingredient (lb)</th>
<th>Protein</th>
<th>Calcium</th>
<th>Fat</th>
<th>Calories</th>
<th>Cost per Pound</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Corn</td>
<td>10</td>
<td>12</td>
<td>35</td>
<td>15</td>
<td>750</td>
<td>$0.65</td>
</tr>
<tr>
<td>2. Soybeans</td>
<td>25</td>
<td>35</td>
<td>15</td>
<td>60</td>
<td>1,000</td>
<td>$0.50</td>
</tr>
<tr>
<td>3. Fish Meal</td>
<td>20</td>
<td>40</td>
<td>10</td>
<td>150</td>
<td>1,500</td>
<td>$0.45</td>
</tr>
<tr>
<td>4. Oats</td>
<td>15</td>
<td>18</td>
<td>15</td>
<td>12</td>
<td>1,100</td>
<td>$0.45</td>
</tr>
<tr>
<td>5. Limestone</td>
<td>6</td>
<td>60</td>
<td>50</td>
<td>5</td>
<td>300</td>
<td>$0.40</td>
</tr>
</tbody>
</table>

The minimum daily nutrient and calorie requirements per animal are provided in the following table:

<table>
<thead>
<tr>
<th>Nutrient (mg)</th>
<th>Feed Mix</th>
<th>Protein</th>
<th>Calcium</th>
<th>Fat</th>
<th>Calories</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cattle</td>
<td>25</td>
<td>200</td>
<td>120</td>
<td>5,500</td>
</tr>
<tr>
<td></td>
<td>Horse</td>
<td>15</td>
<td>150</td>
<td>100</td>
<td>3,500</td>
</tr>
<tr>
<td></td>
<td>Chicken</td>
<td>10</td>
<td>15</td>
<td>10</td>
<td>150</td>
</tr>
</tbody>
</table>

Based on their own crop output, their storage space, and the number of different livestock they have, the Deeres estimate that they have at most 5 pounds of corn, 7 pounds of soybeans, 3 pounds of fishmeal, 8 pounds of oats, and 3 pounds of limestone nutrient mix available each day for their feed mixture. Formulate a linear programming model for this problem and solve the model using the computer.
Blending Problem

Sunbelt Bater Oil and Gas Company produces and sells high-octane and low-octane grades of unleaded gasoline. It produces these two products by blending raw gasoline stocks purchased on the open market. It has two raw gasoline stocks in storage with octane levels as follows:

<table>
<thead>
<tr>
<th>Raw Gasoline Stock</th>
<th>Gallons</th>
<th>Octane Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1,000,000</td>
<td>70</td>
</tr>
<tr>
<td>2</td>
<td>1,000,000</td>
<td>100</td>
</tr>
</tbody>
</table>

Demand for each of the two grades of unleaded gasoline is directly related to the money spent on advertising. Each dollar spent on advertising creates a demand for high-octane gas of 8 gallons and demand for low-octane gas of 10 gallons. The company advertises that high-octane gas has an average octane level of at least 95 and low-octane gas has an average octane level of at least 80. A gallon of high-octane gas is sold for $1.10, and a gallon of low-octane gas is sold for $0.90. The company wants to determine how many gallons of high- and low-octane unleaded gasoline to produce, and how much to spend on advertising in order to maximize total revenue. Formulate a linear programming model for this problem and solve the model using the computer.